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Module 3: The Moon Part 2

Ever wondered how scientists figure out the sizes of craters and mountains on the Moon? In this section, you'll learn how to do just that using the **Pythagorean Theorem** and **trigonometric ratios**! By applying these math tools, you'll be able to measure the lengths of different lunar features, just like real astronomers do. And the best part? The same rules can be used to measure all kinds of objects in space, from distant planets to massive asteroids! So, get ready to sharpen your math skills and explore the Moon in a whole new way!

Pythagorean Theorem Refresher

The **Pythagorean Theorem** is a special rule that helps us find missing side lengths in **right triangles**. It states that in any right triangle, the sum of the squares of the two shorter sides (**legs**) is equal to the square of the longest side (**hypotenuse**).

This is written as:

$$a^2 + b^2 = c^2$$

Where:

- **a** and **b** are the legs of the triangle.
- **c** is always the **hypotenuse** (the longest side, opposite the right angle).

For example, if a right triangle has legs of **3 feet** and **4 feet**, we can find the hypotenuse:

$$3^2 + 4^2 = c^2$$

$$9 + 16 = c^2$$

$$25 = c^2$$

$$c = \sqrt{25}$$

$$c = 5$$

So, the **hypotenuse is 5 feet**!

The Pythagorean Theorem is useful for finding missing distances not just in triangles, but in **real-world applications** like **measuring objects on the Moon** and even **finding distances in space**. You'll also use **trigonometric ratios** along with this theorem to calculate lengths of different lunar features!

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Refresher on Trigonometric Ratios for Right Triangles

Trigonometry is used to find missing sides and angles in right triangles by applying specific ratios. In a right triangle:

- The hypotenuse (h) is always the longest side, located opposite the right angle (90°).
- The opposite (o) side is across from the given angle (θ).
- The adjacent (a) side is next to the given angle (θ) but is not the hypotenuse.

The Three Main Trigonometric Ratios

1. Sine (sin) = opposite / hypotenuse
2. Cosine (cos) = adjacent / hypotenuse
3. Tangent (tan) = opposite / adjacent

A useful way to remember these is with the mnemonic "SOH-CAH-TOA":

- Sine = Opposite / Hypotenuse
- Cosine = Adjacent / Hypotenuse
- Tangent = Opposite / Adjacent

Example: Finding a Missing Side

Consider a right triangle where:

- The given angle θ = 40°
- The hypotenuse = 15 meters
- The goal is to find the opposite side (x)

Since the opposite side and hypotenuse are involved, the sine function is used:

$$\sin(40°) = \frac{x}{15}$$

Solving for x:

$$x = 15 \times \sin(40°)$$

Using a calculator:

$$x \approx 15 \times 0.6428$$

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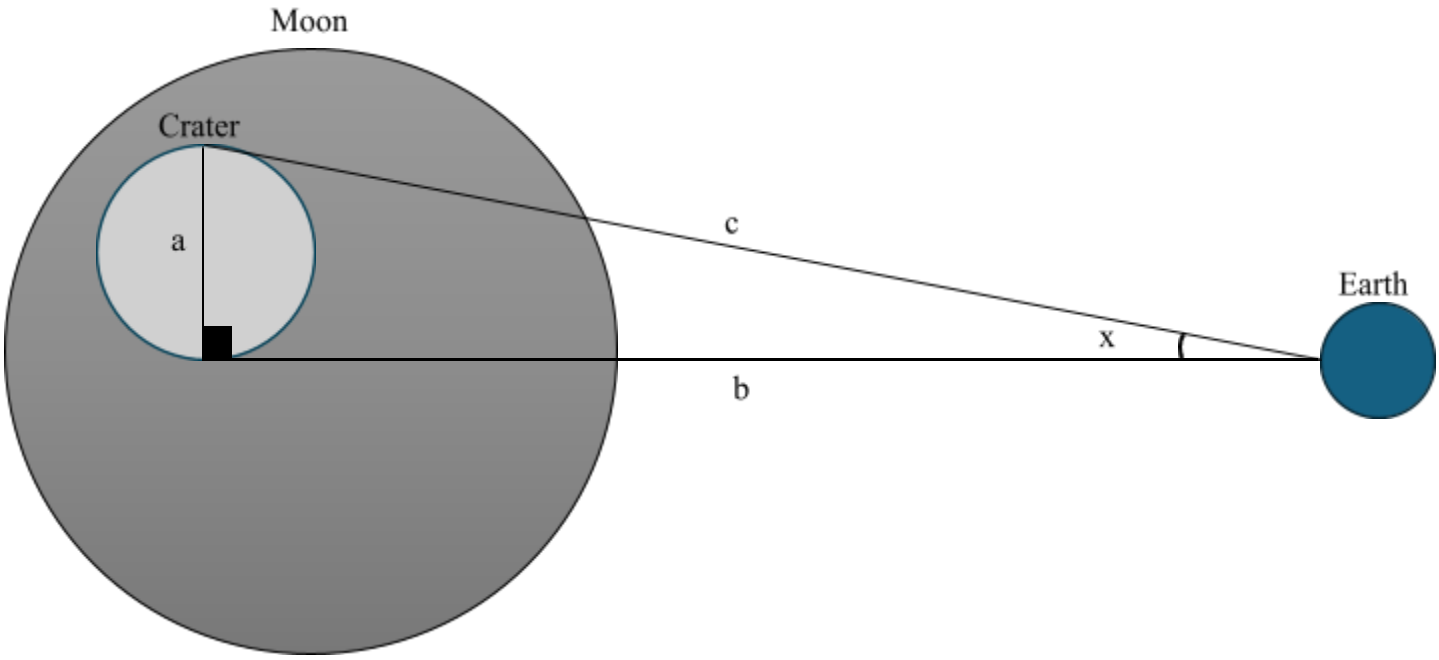
$x \approx 9.64$ meters

The length of the opposite side is approximately 9.64 meters.

Applying Trigonometry to the Moon and Space

The same trigonometric rules can be used to measure the sizes of craters, mountains, and other objects on the Moon. These methods are also applied in astronomy to determine distances to planets, stars, and other celestial bodies. Understanding these principles provides a foundation for measuring large-scale distances beyond Earth.



Example Question: Finding the Length of 'a' Using the Pythagorean Theorem and Trigonometric Ratios

Astronomers on Earth are studying a crater on the Moon. They take measurements and determine that:

- The distance from **Earth to the Moon** (b) is **384,400 km**.
- The angle **x** between the line from Earth to the crater and the direct line from Earth to the Moon is **5°**.

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Step 1: Finding the Hypotenuse (c) Using the Cosine Function

We use the **cosine function** because we know the adjacent side (**b**) and need to find the hypotenuse (**c**):

$$\cos cos\ (x)\ =\ \frac{b}{c}$$

Solving for **c**:

$$c\ =\ \frac{b}{cos(x)}$$

Substituting values:

$$c\ =\ \frac{384400}{cos(5°)}$$

$$\text{c} = 385{,}900.36\text{km}$$

So, the hypotenuse is **385,900.36 km**.

Step 2: Finding 'a' Using the Pythagorean Theorem

Now that we have **c**, we use the **Pythagorean Theorem**:

$$a^2 + b^2 = c^2$$

Solving for **a**:

$$a\ =\ \sqrt{c^2 - b^2}$$

Substituting Values:

$$a\ =\ \sqrt{385900.36^2 - 384400^2}$$

$$a\ =\ \sqrt{1.128\times10^{11} - 1.478\times10^{11}}$$

$$a\ =\ \sqrt{1.496\times10^{9}}$$

$$a\approx 13,435.93\ km$$

So, the length of **a** is **13,435.93 km**.

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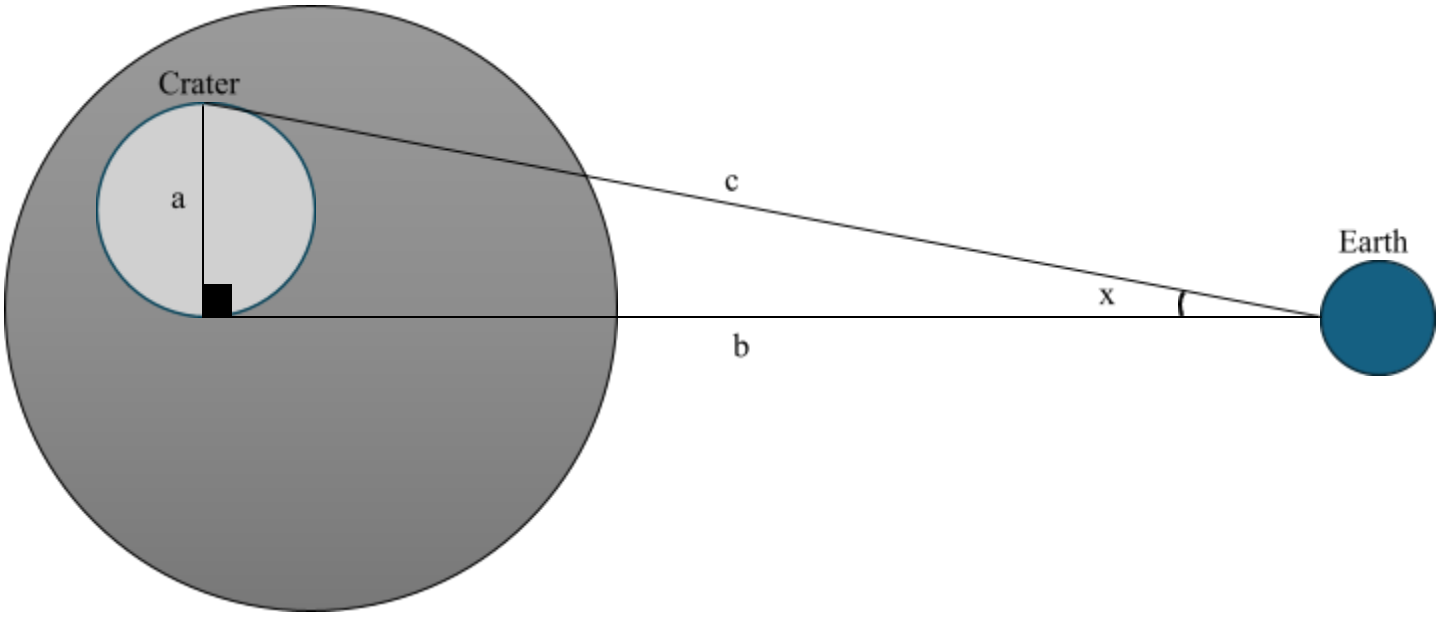

Finding the Length of 'a' Using Trigonometry and the Pythagorean Theorem

Astronomers are studying a crater on the Moon and take measurements from Earth. They determine the following:

- The **distance from Earth to the Moon** (b) is **384,000 km**.
- The **angle (x) between the direct Earth-Moon line and the line to the crater** is **6°**.

1. **Use the cosine function** to calculate the **hypotenuse (c)**, which represents the distance from **Earth to the crater**.
    - Use the formula:

$$\cos cos\ (x) \ = \ \frac{b}{c}$$

    - Solve for **c**.
2. **Use the Pythagorean Theorem** to calculate the **length of 'a'**, which represents the vertical distance from the Moon's surface to the top of the crater.
    - Use the formula:

$$a^2 + b^2 = c^2$$

    - Solve for **a**.

Express your final answers in kilometers and round to two decimal places.

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Sources

Pythagorean Theorem:

https://youtu.be/nCD-bAEbB3I?list=TLGGS15ntUJDmC0wNDAzMjAyNQ

Trigonometry refresher:

https://www.youtube.com/watch?v=GtpplO7xdqM