Date: _________ Name: __________

Class: __________

Module 3: Our Moon Part 2

Ever wondered how scientists figure out the sizes of craters and mountains on the Moon? In this section, you'll learn how to do just that using the Pythagorean Theorem and trigonometric ratios! By applying these math tools, you'll be able to measure the lengths of different lunar features, just like real astronomers do. And the best part? The same rules can be used to measure all kinds of objects in space, from distant planets to massive asteroids! So, get ready to sharpen your math skills and explore the Moon in a whole new way!

For some extra help for this module see:

Pythagorean Theorem:https://youtu.be/nCD-bAEbB3I?list=TLGGS15ntUJDmC0wNDAzMjAyNQ

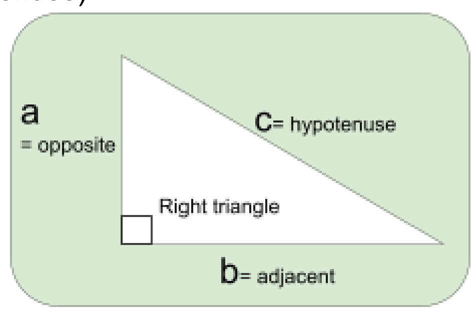
Trigonometry refresher: https://www.youtube.com/watch?v=GtpplO7xdqM

Pythagorean Theorem

The Pythagorean Theorem is a special rule that helps us find missing side lengths in right triangles. It states that in any right triangle, the sum of the squares of the two shorter sides (legs) is equal to the square of the longest side (hypotenuse).

This is written as:

$$a^2 + b^2 = c^2$$



[Image: EA Hyde, AICO]

Where:

- a and b are the **opposite** and **adjacent** legs of the triangle.
- c is always the **hypotenuse** (the longest side, opposite the right angle).

For example, if a right triangle has legs of 3 feet and 4 feet, we can find the hypotenuse:

$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c$$
$$5 = c$$

<u>So, the hypotenuse is 5 feet!</u>

The Pythagorean Theorem is useful for finding missing distances not just in triangles, but in real-world applications like measuring objects on the Moon and even finding distances in space. You'll also use trigonometric ratios along with this theorem to calculate lengths of different lunar features!

Date: __________ Name: ___________

Class: ___________

Refresher on Trigonometric Ratios for Right Triangles

Trigonometry is used to find missing sides and angles in right triangles by applying specific ratios. In a right triangle:

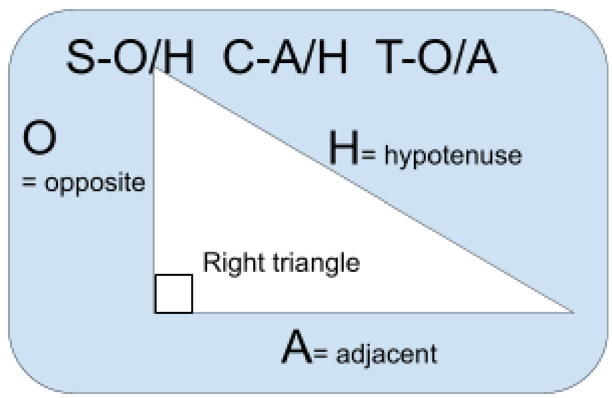
- The hypotenuse (H) is always the longest side, located opposite the right angle (90°).
- The opposite (O) side is across from the given angle (θ).
- The adjacent (A) side is next to the given angle (θ) but is not the hypotenuse.

The Three Main Trigonometric Ratios

1. Sine (sin) = opposite / hypotenuse =O/H
2. Cosine (cos) = adjacent / hypotenuse = A/H
3. Tangent (tan) = opposite / adjacent = O/A

A useful way to remember these is with the mnemonic "SOH-CAH-TOA":

- **S**ine = **O**pposite / **H**ypotenuse
- **C**osine = **A**djacent / **H**ypotenuse
- **T**angent = **O**pposite / **A**djacent



[Image credit: EA Hyde, AICO]

Example:
Finding a Missing Side of a Right Triangle

Consider a right triangle where:

- The given angle θ = 40°
- The hypotenuse = 15 meters
- The goal is to find the opposite side (x)

Since the opposite side and hypotenuse are involved, the sine function is used:

***S**ine = **O**pposite / **H**ypotenuse*

$$sin(40^o) = \frac{x}{15}$$

$$sin(40^o) \times 15 = x$$

$$0.6428 \times 15 = x$$

$$9.64 \approx x \text{ (meters)}$$

The length of the opposite side is approximately **9.64 meters.**

This means the Pythagorean Theorem can also be written as:

$$O^2 + A^2 = H^2$$

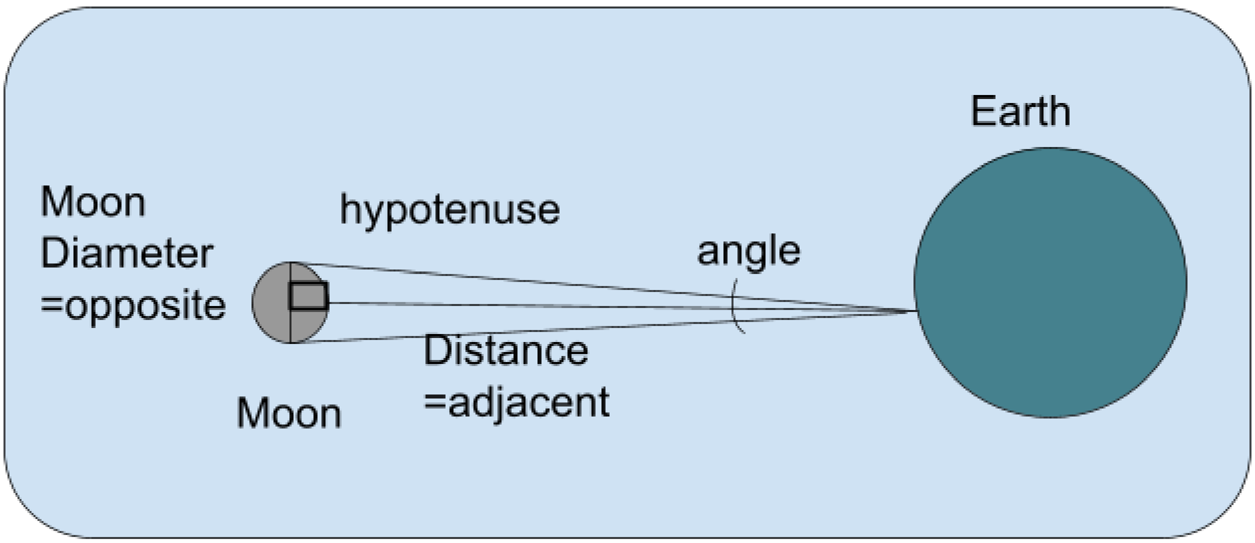
Using the SOH-CAH-TOA naming. With both of these tools we can do a lot!

Date: _________   Name: __________

Class: __________

Applying Trigonometry to the Moon and Space

The same trigonometric rules can be used to measure the sizes of craters, mountains, and other objects on the Moon, or even the size of the Moon itself. These methods are also applied in astronomy to determine distances to planets, stars, and other celestial bodies. Understanding these principles provides a foundation for measuring large-scale distances beyond Earth. This diagram below does not show true distance as it would make it hard to see the size of the moon, however, you will use the real distances in the example with the formulas mentioned.



[Image credit: EA Hyde, AICO]

If you are wondering why we do not use the real size of the Earth and Moon with their real distance when drawing the angles, see below!

[Image credit: NASA, JPL/Caltech Earth and moon to scale]

Actually the moon is quite far away! The distance in the NASA figure is also a bit different than what we will use for our problems as the Moon changes its distance from Earth just a little bit as it orbits, but that is an investigation for another time.

Date: _________   Name: __________

Class: __________

Worked Example: The size of the moon in the sky is often said to be about ½ of a degree. This means that if you hold your arm outstretched you can block the full moon with the end of your pinky finger (try it!). Since we know the angle, let's try 2 ways to solve for the size (diameter) of our Moon.

We will use the following numbers

- The average distance from Earth to the Moon (our adjacent side) is **385,000 km.**
- The angle between the line from Earth to Moon edge and the direct line from

Earth to the Moon's is ¼ a degree, to get the full angle from edge-to edge, use 2x1/4 = **0.5° = 0.5 degrees.**

Tip: make sure your calculator is set to degrees and not radians when doing calculations!

First method: Finding the Length of 'a' Using the Pythagorean Theorem and Trigonometric Ratios.

Step 1: Examine the function! To solve for a in the pythagorean theorem we need the hypotenuse and the adjacent sides. Since we already know the adjacent side, if we solve for the hypotenuse first we can use the pythagorean theorem.

Since we have the distance b (the adjacent side) and the angle we can solve for the Hypotenuse (c) using the Cosine Function:

Cosine = **A**djacent / **H**ypotenuse

$$cos(5^o) = \frac{distance}{h}$$

$$h = \frac{distance}{cos(0.5^o)}$$

$$h = \frac{384400}{cos(0.5^o)}$$

$$h = 385014.66$$

So, the hypotenuse is 385,014.66 km.

Step 2: Finding 'a' Using the Pythagorean Theorem

Date: __________ Name: ___________

Class: ___________

Now that we have the hypotenuse, we use the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$
$$a = \sqrt{(b^2 - c^2)}$$
$$a = \sqrt{(385014.66^2 - 385000^2)}$$
$$a = \sqrt{(1.48236 \times 10^{11} - 1.48225 \times 10^{11})}$$
$$a = \sqrt{(1.289 \times 10^7)}$$
$$a \approx 3360.03\ km$$

So, the diameter (or size) of the Moon is about 3360 km according to our angle. From accurate modern measurements we know that the true diameter is 3476 km, so our estimate got us very close to the real result! Let's try a second method to solve this problem and see what we get.

Second method: Use the tangent function to get the opposite side of the triangle (the diameter of the Moon), in only 1 step instead of 2.

We are still looking at the size of the moon, so our average distance from Earth to the Moon (our adjacent side) is **385,000 km** and the angle is **0.5° = 0.5 degrees.**

Since we have the distance (the adjacent side) and the angle we can solve for the opposite site (The Moon's diameter) directly using the Tangent Function:

Tangent = **O**pposite / **A**djacent

$$tan(0.5^o) = \frac{distance = O}{adjacent = A}$$
$$A = Moon\ Diameter = \frac{distance}{tan(0.5^o)}$$
$$A = \frac{385000\ km}{tan(0.5^o)}$$
$$A = 3359.84\text{km}$$

This is very close to our estimate from the first method! In fact, it is likely just rounding errors that form the difference. Which method do you prefer?

Date: _________ Name: __________

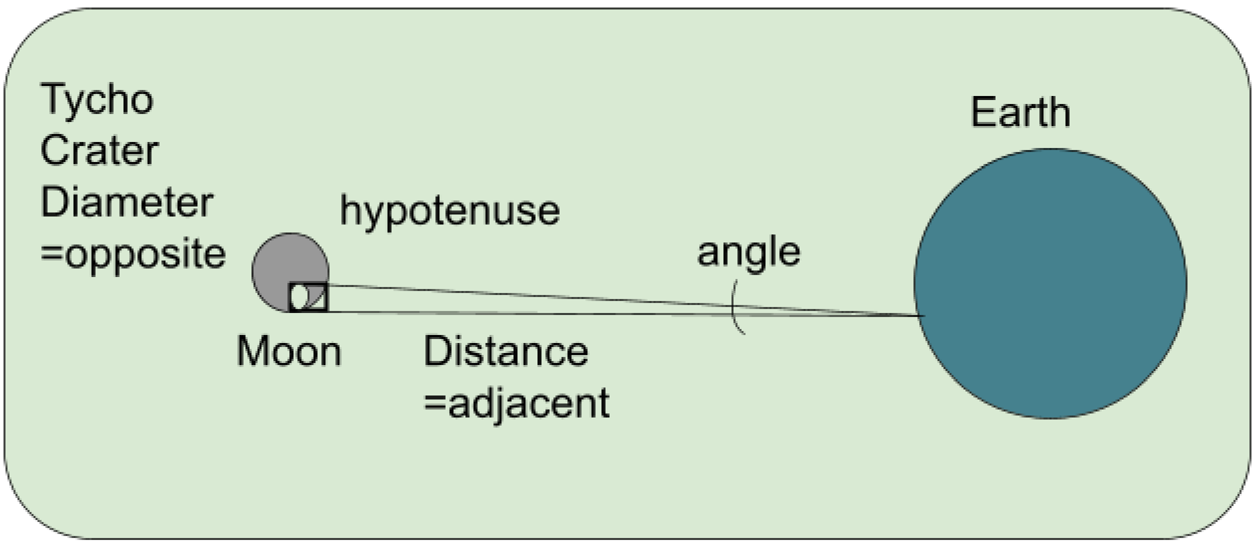
Class: __________

Student's Problem:

Tycho is one of the biggest and brightest craters on our Moon that we can easily see. It features striking white outlines, and is visible to humans all over the world. Instead of getting the diameter of the Moon you are going to measure the size of Tycho crater from its largest angle (it will be much smaller than for the whole Moon!). We will measure from the edges of the large ray spokes of Tycho crater which are known to stretch out to 1500 km. Can you get close to that value with your solution?

Solve the right triangle problem using the following numbers

- The average distance from Earth to the Moon (our adjacent side) is 385,000 km.
- The angle between the line from Earth to Tycho craters edge and the direct line from Earth to the other edge of Tycho crater is 0.225° = 0.225 degrees.



[Image credit: EA Hyde, AICO]

Q1: Show your work and use method 1 or 2 from the worked example

Q2: The inside area of Tycho crater is 85 km in diameter, however our angle measures from the outer edge of the rays at 1500 km. How close did you get to 1500 km?

Q3: Can you imagine how much more difficult it would be to measure the smaller craters with this technique?